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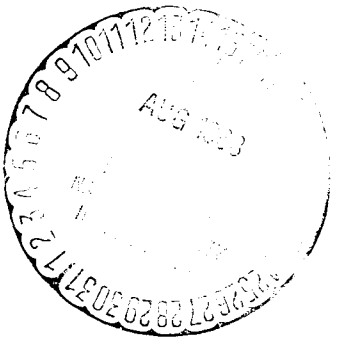

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DEPENDENCE BETWEEN THE STROUHAL AND REYNOLDS NUMBERS IN TWO-DIMENSIONAL FLOW PAST A CIRCULAR CYLINDER

S. G. Popov

ABSTRACT: The relationship between the Strouhal and Reynolds numbers in two-dimensional flow around circular cylinders and the structure of the vortical wake behind the cylinders are investigated for the cases of stationary and vibrating cylinders for a wide range of Reynolds numbers including the subcritical, critical, and supercritical zones. New data are compared and supplemented with old data and theories of the real function S(R) are derived for a circular cylinder. Separate data from studies on the nature of the oscillators of smoke stacks transverse to the wind are collected and processed.

The structure of the flow in the wake behind a circular cylinder around /156[1]
which a fluid or gas is flowing perpendicular to its axis has been studied experimentally as a function of the parameters, by many investigators and by various methods[2]. In this case, discrete vortices, including several pairs which form a sort of two-way road with a checkered arrangement, have been discovered in a wide range of Reynolds numbers $R = Vd/\nu$; the frequency of vortices n_1 appeared, in many cases, to be a linear function of the velocity V of an unperturbed flow. Therefore the dependence of the function of Strouhal numbers $S = n_1 d/V$ on the value of R and the method of its determination was sought. The results of several authors turned out to differ but little in the subcritical part of the curve of $S_x(R)$ for a cylinder; the critical and supercritical regions, however, are either contradictory or inaccurate. Therefore, we will compare known experimental data, supplement these by new data, and derive theories concerning the real function S(R) for a circular cylinder. The explanation of this problem is essential for the theory of the adjacent flow behind a profile and for calculations of the oscillations in the flows in technical equipment in which circular cylinders are the basic elements (pipes, conductors, antennas, high smoke stacks, etc.).

Let us first examine experimental data on stationary cylinders in a flow. The well-known function S(R) is illustrated in Figure 1 by a solid line in the same form as shown in the book by Goldstein [3], according to the experiment of Relf and Simmons [4]. Similar results were obtained in several other past experiments; they were represented by Lehnert [5] as a band with a width of up to 0.04 S in the range $1.5 \cdot 10 < R < 8 \cdot 10^4$. We may assume on the basis of all

[1]Numbers in the margin indicate pagination in the foreign text.
[2]Prior to Strouhal [1], the origins of various of these investigations have been traced ultimately to Leonardo da Vinci.

these data that

$$S \approx 0.2 \tag{1}$$

in the range

$$2.5 \cdot 10^2 < R < 2.5 \cdot 10^5 \tag{2}$$

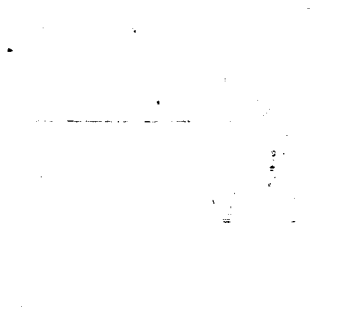

Figure 1.

As stated in [3], the values of S to the right of interval (2) increase at first up to 0.3 and more, and then, with further increase in the R numbers, all regular periodic variations in pressure and velocity in the wake behind the circular cylinder disappear. The analogous tendencies toward an increase in the S numbers in the range

$$2 \cdot 10^5 < R < 10^6 \tag{3}$$

are also represented in the works of Drescher [6], Fugino [7], Itaya and Yasuda [8] for stationary cylinders.

However, the known studies of vortices behind a stationary circular cylinder have produced different results. First of all, in the relatively old work of G. I. Petrov and R. I. Shteynberg [9], the approximate relationship (1) was obtained in the range

$$7 \cdot 10^4 < R < 3 \cdot 10^5$$

(1, Figure 1); as the author [9] pointed out, when $3 \cdot 10^5 < R < 8 \cdot 10^5$ they did not observe periodic oscillations with amplitudes comparable to those recorded in the first interval. The opinion was also expressed in [9] that the experimental points determining the steep rise in the right-hand section of the curve of Relf and Simmons are random and are attributed to the method for measuring the pulsations. Furthermore, Yu. G. Zakharov and L. Kh. Blyumina [10] investigated S(R) for stationary and oscillating cylinders in the range

$$1.7 \cdot 10^5 < R < 1.33 \cdot 10^6 \tag{4}$$

and for stationary cylinders they obtained the values of S according to the frequencies n_2 (2, Figure 1), which were distributed on both sides of straight line (1), even in the critical zone. Gerrard also obtained S(R) in the form of (1) in the range $3.8 < \log R < 5.3$ without an increase in the S numbers in the right-hand portion of this range [11]. For stationary cylinders with supercritical Reynolds numbers in the range $10^6 < R < 5 \cdot 10^6$, Fung [12], used /157
the experimental results of Delany and Sorensen [13], where the Strouhal numbers was also found scattered in the range of S from 0.32 to 0.46, as calculated for the frequencies n_1 in a narrow range of our numbers, (3, Figure 1). However, the more careful investigations of Roshko [14] for even higher values of R yielded S numbers which differed sharply from the data of [13], as seen in Figures 1, 4; furthermore, Roshko noted that for the corresponding values of R he was not able to record those vortical frequencies n_1 and the corresvalues of S which were given in [13][1].

Thus, according to the data of the basic work cited herein, it is possible to make a general conclusion regarding stationary cylinders: For values of R less than critical R_*, condition (1) is approximately satisfied in the interval (2); but for critical and supercritical values of R we know the data concerning the deviation of the values of S from that of (1) (see Figure 1); and these data give rise to serious doubts.

In the case of oscillation of the cylinders transverse to the flow, the function S(R) has a more definite form than for stationary cylinders throughout almost the entire range of R numbers. Actually, even for $R < R_*$, several investigators used the method for determining the S numbers according to the frequency n_2 of the transverse oscillations of circular cylinders (rods) in a flow at maximum amplitude (see, for example, the works of Kruger [15], Thom [16], Tyler [17]; moreover, the approximate condition (1) was also obtained. Fugino [7] found $S = 0.18$ for an oscillating cylinder as determined by the frequency n_2 in the same interval (3) in which he found the ascending part of the curve of S(R) in the same work by the measurements of the frequencies of

[1] This discrepancy is apparently attributable to the experimental technique, as was assumed long ago [9]: Thus, in the tests of [14] the thermoanemometer used for measuring the frequencies was located at a distance of 7.3 diameters from the cylinder, whereas, in [13] (as in [4] it was located at a distance of 1 to 2 diameters from the cylinder. Since the vortices are formed at a distance of several diameters from the cylinder by the smaller high-frequency vortices, it was therefore obviously possible to record large S numbers in [13]. Furthermore, when the thermoanemometer is located on the axis of the stream, it is possible to record the vortices of both lanes of the road, i.e., the double frequency of the vortices.

the vortices n_1 behind a stationary cylinder (Figure 1) [1].

Finally, we have collected and processed here the separate data from studies on the nature of the oscillations of smoke stacks transverse to the wind and have represented them in Figure 2 where the points 1 are those of Den Hartog [9, 20], 2 - Hausner [2], 3 - Ozker [22], and also the data of the vibrations of the cylinders, 4 - [10] and 5 - [7], by deriving S(R) on the basis of these values of n_2. Here again the grouping of all points is clear, particularly as regards (1) in the range

$$5 \cdot 10^5 < R < 10^7 \tag{6}$$

where the oscillations were observed under the conditions $n_2 \approx n_3$ according to the data of [19-22].

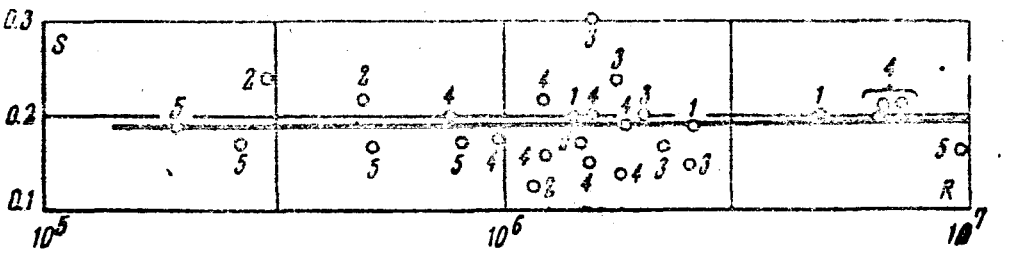

Figure 2.

[1]Detailed investigations of the relationship between the frequency of vortices flowing from a stationary cylinder n_1, the vibration frequency of the cylinder n_2, the natural frequency n_3 of vibrations and the velocity of the flow V (also for $R < R_*$) are discussed in [16] (see also [10]). It was firmly established here that in addition to the Karman vortices with frequency n_1, which are linear functions of V through (1), another vortex emerges during the vibrations of the cylinder in each extreme position of its transverse stand. The greatest amplitudes of vibrations are found when the conditions

$$n_2 \approx n_3 \approx n_1 \tag{5}$$

are (approximately) satisfied.

As the velocity V increases within a certain range, the value of n_1 may become noticeably greater than n_2 and n_3, although n_2 also increases with V but very slowly; in this case the amplitudes drop to almost 0 while the velocity V does not attain the value at which the approximate condition (5) is satisfied with the next natural frequency n_3; then the amplitudes once more become high; and so on.

The means for visualizing the vortices and for determining n_1 were not employed in [19-22]; data concerning the frequencies n_1 which satisfy (4) are used in [10]. Since the mechanism for exciting the vibrations of the smoke stacks is the same as that for $R < R_*$, the transverse vibrations of the specified smoke stacks must be accompanied by the expulsion of the vortices in the extreme positions of the range. Therefore, it will also be satisfied for supercritical $R > R_*$.

The results of the experiment of Nakagawa, et. al [23] in the range /158

$$1.5 \cdot 10^5 < R < 1.5 \cdot 10^6 \qquad (7)$$

for four vibrating round cylinders 0.5 meters in diameter, of which cylinder 1 is smooth, cylinders 2 and 3 have helical wire windings of 4 and 8 wires respectively, and cylinder 4 is smooth and located behind a turbulence lattice, may be very interesting supplements to the data described above. The last three cylinders produced oscillations with the highest amplitudes for the values of S according to (1) and cylinder 1 yielded values of S scattered from 0.15 to 0.3, where all S numbers were computed according to frequency n_2. The corresponding S numbers according to the values of n_1 are illustrated in Figure 3 for vibrating cylinders 2, 3, and 4, and the greatest scattering was observed for cylinder 1 (Figure 4).

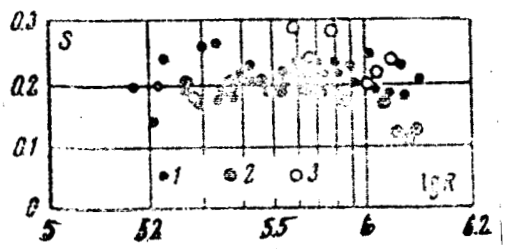

Figure 3

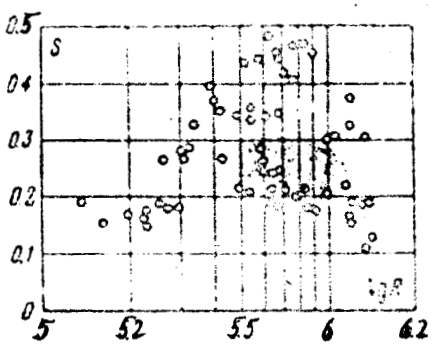

Figure 4

Thus, significant scattering was observed in the critical region, both of vibration frequencies n_2 and frequencies n_1 of the vortices behind the vibrating cylinder only for the smooth model, where the points (line) of breakaway is unstable where the supercritical conditions exists as a result of the wire winding (cylinders 2 and 3), or because of the turbulent lattice (cylinder 4), the scattering of n_2 and n_1 becomes slight and all values of S, even in the critical range, satisfy the approximate condition (1). But even for smooth

cylinder 1 it is possible to assume (condition 1) to some extent if we take into account the results for n_2 and the distribution of the points of n_1 through interval (7).

Now, by generalizing the data from the cited literature for intervals (3), (4), (7), and (6), we will have the approximate conditions (1) and (5) in the range

$$2.5 \cdot 10^2 < R < 10^7 \qquad (8)$$

for vibrating cylinders (rods or wires).

The vibrations of a cylinder (of a rod or wire) of circular profile thus stabilize the formation of vortices caused by them, just as in any self-oscillating system.

Relations (1) and (5) may be used for the calculation of the vibrations listed in the introductory section, for R in range (8), and apparently for large R numbers. Tne calculations of the vibrations of smoke stacks during typhoons [24], which had been confirmed by experience, are the simplest examples.

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